MINIMAX ESTIMATOR & EMPIRICAL BAYES ESTIMATOR

1. Define: (i) Admissible estimator, (ii) Descision function, (iii) Loss function, (iv) Risk function. [Saleh & Rohatgi: Page-424]

2. Explain the way of obtaining minimax estimator along with example.

3. Let $X \sim b(1, p), p \in \theta = \left\{\frac{1}{4}, \frac{1}{2}\right\}$ *and* $A = \{a_1, a_2\}$. Let the loss function be defined as follows.

	a_1	a_2
$p_1 = \frac{1}{4}$	1	4
$p_2 = \frac{1}{2}$	3	2

Find the minimax solution. [Saleh & Rohatgi: Example 04: Page-425]

4. Define bayesian risk. [Saleh & Rohatgi: Definition 05: Page-426]

5. Let $X \sim b(n, p)$ and $L(p, \delta(x)) = [p - \delta(x)]^2$. Let $\pi(p) = 1$ for $0 < p < 1$ be the a priori PDF of p. Then check whether the bayes estimator is the minimax estimator. [Saleh & Rohatgi: Example 05: Page-428]

6. Let$X \sim N(\mu, 1)$, and let the a priori PDF of μ be $N(0,1)$. Also let $L(p, \delta(x)) = [p - \delta(x)]^2$. Then check whether the bayes estimator is the minimax estimator. [Saleh & Rohatgi: Example 06: Page-428]

7. Let $\{f_\theta : \theta \in \boldsymbol{\theta}\}$ be a family of PDFs (PMFs), and suppose that an estimator δ^* of θ is a Bayes estimator corresponding to an a priori distribution π on $\boldsymbol{\theta}$. If the risk function $R(\theta, \delta^*)$ is constant on $\boldsymbol{\theta}$, then δ^* is a minimax estimator for θ. [Saleh & Rohatgi: Theorem 02: Page-435]

8. Let $X \sim b(n, p), 0 < p < 1$. We seek a minimax estimator of p of the form $\alpha X + \beta$, using the squared-error loss function. [Saleh & Rohatgi: Example 15: Page-436]

EMPIRICAL BAYES ESTIMATOR

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9. What do u mean by empirical Bayes estimator ? Explain with an example . [lehmann-and-george-casella .Page 262]

10. Let $X_k \sim binomial(p_k\ , n) and\ p_k \sim beta(a, b)$

   $Find\ emperical\ bayes\ estimator\ of\ p_k$.[lehmann-and-george-casella. Page 263.Example 6.2]

11. $If\ X_1\ , X_2\ , \ldots \ldots X_p\ \ has\ the\ density\ p_\eta(x) = e^{\sum_{i=1}^{p} \eta_i x_i - \lambda(\eta)} h(x)\ and\ \eta\ has\ prior\ density\ \pi(\eta|\lambda), Suppose\ \hat{\lambda}(x) is\ the$

   $MLE\ of\ \lambda\ based\ on\ m(x|\lambda). Then\ the\ emperical\ bayes\ estimator\ is$

   $E(\eta_i|x, \hat{\lambda}) = \frac{\delta}{\delta, x_i} \log m\left(x\middle|\hat{\lambda}(x)\right)\ \frac{\delta}{\delta, x_i} \log(h(x))$[lehmann-and-george-casella. Page 265. Theorem : 6.3]

12. Calculate an Empirical Bayes Estimator for the model

   $X_i|\theta \sim N(\theta, \sigma^2)\quad , \Omega \sim \mathrm{N}(0, \tau^2)$[lehmann-and-george-casella. Page 266. Example : 6.4]

13. What is James –Stein estimator ? Write down its properties .[lehmann-and-george-casella. Page 272.]

14. Find the Bayes risk of the James Stein estimator .[lehmann-and-george-casella . Page 273. Example 7.3]

15. Find the poisson hierarchical Bayes estimator using the following model

$$X_i \sim Poisson(\lambda_i)\ \ i = 1,2,3 \ldots \ldots p\ independent\ .$$

   $\lambda_i \sim Gamma(a, b),\ \ i = 1,2,3 \ldots \ldots . p, independent\ , a\ known$ [lehmann-and-george-casella .Page : 268, Example:6.6]